

Impact Parameter Dependent Parton Distributions for a Relativistic Composite System

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Abstract. We investigate the impact parameter dependent parton distributions for a relativistic composite system in light-front framework. We express them in terms of overlaps of light-cone wave functions for a self consistent two-body spin-1/2 state, namely an electron dressed with a photon in QED. The pdfs are distorted in the transverse space for transverse polarization of the state at one loop level.

1 Introduction

Impact parameter dependent parton distributions $q(x, b^\perp)$ [1] have been introduced recently as a physical interpretation of generalized parton distributions (GPDs) in terms of probability densities in the impact parameter space. When the state is transversely polarized, the impact parameter dependent pdf is distorted in the transverse plane [1].

Recently we have done an investigation of the pdfs in the impact parameter space for a relativistic composite system in the light-front framework [2], taking into account the correlation between different Fock components of the light-cone (or light-front) wave function. We take an effective spin $\frac{1}{2}$ system of an electron, dressed with a photon in QED. Such a model is self consistent and has been used to investigate the helicity structure of a composite relativistic system [3]. The state can be expanded in Fock space in terms of light-cone wave functions, which, in this case can be obtained from perturbation theory, and thus their correlations are known at a certain order in the coupling constant. Earlier studies have shown that this gives an intuitive picture of the DIS structure functions and scaling violations [4] and is suitable to address issues related to the spin and orbital angular momentum of the nucleon [5, 6]. Such a state has also been

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used to investigate the twist three GPDs in terms of overlaps of light-cone wave functions [7]. We extend these studies to the impact parameter dependent pdfs and here we report on our main results.

2 Impact Parameter Dependent Parton Distributions

For a transversely localized state [1] the impact parameter dependent pdfs are defined as

$$q(x, b^\perp) = \langle P^+, R^\perp = 0^\perp, \lambda \mid O_q(x, b^\perp) \mid P^+, R^\perp = 0^\perp, \lambda \rangle \quad (1)$$

with

$$O_q(x, b^\perp) = \int \frac{dx^-}{4\pi} \bar{\psi}(-\frac{x^-}{2}, b^\perp) \gamma^+ \psi(\frac{x^-}{2}, b^\perp) e^{\frac{i}{2}xP^+x^-}. \quad (2)$$

Instead of the fermion operator, one can also have a gauge boson operator

$$O_g(x, b^\perp) = \int \frac{dx^-}{4\pi} F^{+\nu}(-\frac{x^-}{2}, b^\perp) F^+{}_\nu(\frac{x^-}{2}, b^\perp) e^{\frac{i}{2}xP^+x^-}, \quad (3)$$

b^\perp is the impact parameter, which is the transverse distance of the active quark from the center of mass. We have taken the light-front gauge, $A^+ = 0$. The transversely localized states are superpositions of helicity eigenstates. It can be shown that $q(x, b^\perp)$ can be expressed as a Fourier transform of the GPD $H_q(x, 0, \Delta^2)$ [1]:

$$q(x, b^\perp) = \mathcal{H}_q(x, b^\perp) = \int \frac{d^2\Delta^\perp}{(2\pi)^2} e^{-ib^\perp \cdot \Delta^\perp} H_q(x, 0, \Delta^2), \quad (4)$$

with

$$H_q(x, 0, \Delta^2) = \int \frac{dz^-}{8\pi} e^{\frac{i}{2}x\bar{P}z^-} \langle P' \uparrow \mid \bar{\psi}(-\frac{z^-}{2}) \gamma^+ \psi(\frac{z^-}{2}) \mid P \uparrow \rangle. \quad (5)$$

We take Δ^2 , the total momentum transfer to be purely transverse.

The helicity eigenstate of an electron dressed with a photon in QED can be expanded in Fock space in terms of 1-body and 2-body light cone wave functions which can be expressed in terms of Jacobi momenta x_i, q_i^\perp . The analytic form of the two-body wave function can be obtained from the light-front eigenvalue equation [4]. The GPD $H_{q,g}(x, 0, \Delta^2)$ can be expressed as an overlap of the light-cone wave functions. Fourier transform *w.r.t* $(\Delta^\perp)^2$ gives $q(x, b^\perp)$. The scale dependency in the impact parameter space comes from the limits of the integration over the intrinsic transverse momenta of the partons [2]. Integrating $H_q(x, 0)$ over x one gets $\int_0^1 dx H_q(x, 0) = F_1(0) = 1$, where $F_1(0)$ is the form factor at zero momentum transfer. At non-zero Δ^\perp , in the limit $x \rightarrow 1$, $H_{q,g}(x, \Delta^2)$ are independent of Δ^\perp . The impact parameter dependent pdf in this limit is a delta function in b^\perp as expected because in this limit the electron carries all the momentum and the transverse width of the impact parameter dependent pdf vanishes [1].

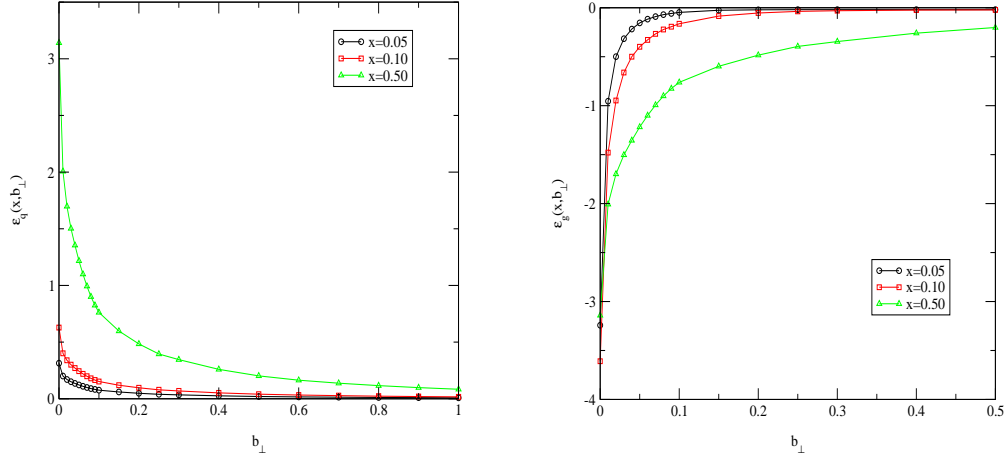


Figure 1. $\mathcal{E}_q(x, b^\perp)$ and $\mathcal{E}_g(x, b^\perp)$ vs b^\perp for three different values of x

The helicity flip part of the matrix element gives

$$\begin{aligned}
& \int \frac{dy^-}{8\pi} e^{\frac{i}{2}P^+y^-x} \langle P + \Delta, \uparrow | \bar{\psi}(\frac{-y^-}{2}) \gamma^+ \psi(\frac{y^-}{2}) | P, \downarrow \rangle = \frac{e^2}{(2\pi)^3} \times \\
& x(1-x)^2(-im)(-i\Delta^1 - \Delta^2) \times \\
& \int \frac{d^2q^\perp}{(q^\perp^2 + m^2(1-x)^2)(q^\perp + (1-x)\Delta^\perp)^2 + m^2(1-x)^2)} \\
& = -\frac{E_q}{2m}(\Delta^1 - i\Delta^2). \tag{6}
\end{aligned}$$

m is the renormalized mass of the electron. There is a similar relation for the gauge boson operator which gives $E_g(x, 0, \Delta^2)$.

Taking the Fourier transform, we get

$$\mathcal{E}_{q,g}(x, b^\perp) = \int \frac{d^2\Delta^\perp}{(2\pi)^2} e^{-ib^\perp \cdot \Delta^\perp} E_{q,g}(x, -(\Delta^\perp)^2). \tag{7}$$

Fig. 1 shows the helicity flip contributions $\mathcal{E}_{q,g}(x, b^\perp)$ as a function of b^\perp for three different values of x . The scale dependence is suppressed in this case. We have plotted for positive b^\perp . $\mathcal{E}_q(x, b^\perp)$ is a smooth function of b^\perp in the range shown and it increases as b^\perp decreases. Also, it increases linearly with x . We have taken the overall normalization $\frac{\alpha}{2\pi} = 1$ in order to study the qualitative behavior and $m = 0.5$. $\mathcal{E}_g(x, b^\perp)$ has a maximum $b^\perp = 0$. $\mathcal{E}_g(x, b^\perp)$ is negative for positive b^\perp and has a negative maximum at $b^\perp = 0$. Like the fermion case, $\mathcal{E}_g(x, b^\perp)$ is larger in magnitude for fixed b^\perp as x increases. As before, we took $\frac{\alpha}{2\pi} = 1$ and $m = 0.5$. When the state is transversely polarized, the derivative of $\mathcal{E}_{q,g}(x, b^\perp)$ gives the distortion of the pdf in the transverse space [1]. The distortion of the distribution in impact parameter space increases as b^\perp decreases and for a given b^\perp the distortion is higher in magnitude for larger values of x . The distortion shifts the distribution $\mathcal{E}_q(x, b^\perp)$ actually towards negative values of b^\perp .

3 Summary

We report on an investigation the impact parameter dependent parton distributions for a relativistic composite system. An ideal framework is based on light-front field theory, where the transverse boosts behave like Galilean boosts and the longitudinal boost operator produces just a scale transformation. We take an effective composite spin 1/2 state, namely an electron dressed with a photon in QED. Using the overlap representation of GPDs in terms of light-cone wave functions, we obtain the scale dependence of the impact parameter dependent pdfs at one loop. The helicity flip part gives the distortion of the pdf in transverse space when the state is transversely polarized.

Acknowledgement. We thank M. Burkardt for valuable discussions. AM thanks the organizers of Lightcone 2004 for a wonderful and stimulating conference. The work of AM has been supported in part by the 'Bundesministerium für Bildung und Forschung', Berlin/Bonn.

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